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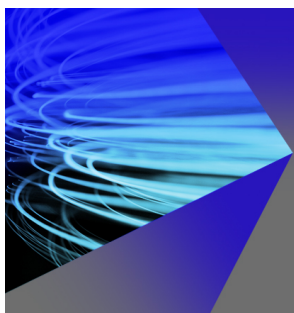
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ABSTRACT

An exhaustive bifurcation diagram of the steady and initially stable periodic solutions arising in double-diffusive convection of an Ar–CO₂ mixture confined in a vertical slot is numerically unfolded in the balanced buoyancy case, with the particularity of taking into account the Soret effect. The consequences of modifying the linear problem including a new term to the equations are first examined. Next, a complex bifurcation diagram of equilibria and periodic orbits is shown. It includes a great amount of extended and localized patterns of convection, retaining or not the initial center symmetry. The stability of the branches of solutions with respect to arbitrary perturbations is analyzed. The study shows that the stable steady flows maximize the mass and heat transport. Two types of stable periodic oscillations have been found coexisting at low Rayleigh number; those attached to the lids, bifurcating from patterns of top and bottom localized vortices, and those bifurcating from patterns of extended vortices. The analysis of the kinetic energy equation at selected bifurcation points out that the power supplied by the temperature and solutal buoyancy terms are the most important, and that they remain always almost in balance.

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I. INTRODUCTION

Many natural phenomena concerning oceanography, astrophysics, geophysics, geology, etc., and industrial applications such as crystal growth, storage of nuclear wastes, manufacturing processes, etc., involve convection driven by thermal and solutal gradients acting either in the same or in transversal directions.

One of the most intricate cases of steady convection under the point of view of the bifurcation and stability analysis is perhaps that in which the initial buoyancy forces due to externally set thermal and solutal gradients are equal, of opposite effects, and normal to the gravity. In this situation a conduction state exists, whose instability is due to the difference of the thermal and solutal diffusivities.^{9,15} From now on it will be called balanced buoyancy case (BBC), and it is the subject of the present study.

The primary and secondary flows arising from the conduction state in the BBC were studied previously in vertical channels,^{2,3,5,13,15} slots,⁵ and rectangular cavities^{5,15} of different periodicity or aspect ratios. All of them assume isothermal and permeable vertical boundaries and neglect the Soret and Dufour effects. The former accounts for the generation of concentration gradients due to the temperature

gradients established in the mixture, while the latter quantifies the generation of temperature gradients due to those induced by the migration of the components.

The critical thermal Rayleigh number, Ra_c^T , of the first instability of the conduction state in periodic vertical layers was determined, after several approximations, by Ghorayeb and Mojtabi.⁵ They found that the critical parameter fulfilled $Ra_c^T[Le^{-1} - 1] = 6509$, for the primary steady bifurcation, with critical wave number 2.53. Here Le is the Lewis number defined in Sec. II. Notice that for $Le = 1$, the conduction state is always stable. There is no Ra_c^T because the difference of the temperature and the concentration equations becomes a diffusion equation¹⁵ in the absence of the Soret and/or Dufour effects.

The steady dynamics arising from this double-diffusive instability is very rich, the primary bifurcation being subcritical.^{5,15} It includes center-symmetric steady patterns filling the domain, or localized either in the center of the cavity (sometimes named convectons) or near the lids (sometimes named anticonvectons^{1–3}). Moreover, symmetry-broken (SB) steady flows with the vortices placed anywhere were already found a few decades ago by means of direct numerical simulation.⁵

The bifurcation diagrams of the steady solutions for mixtures of Prandtl number $Pr = 1$, and Lewis number $Le^{-1} = 11$ in vertical slots of periodic vertical boundary conditions were computed by using numerical continuation methods for $\Gamma = 7.5$ and $\Gamma = 30$ in Bergeon and Knobloch.^{2,3} All the bifurcations were of revolution due to the invariance of the solutions by vertical translations. The stability of the branches of solutions was only analyzed with respect to perturbations maintaining the reflection symmetry of the equations. The authors found, in the first paper, a multiplicity of interconnected, extended, and localized steady states arising from three primary pitchfork bifurcations (PBs) of the conduction state. Half of them were due to the invariance of the solutions by vertical translations $\Gamma/2n$, where n meant the number of pairs of vortices within the enforced spatial period. These solutions were spatially periodic. The localized states were born at secondary transcritical bifurcations (TBs). They were theoretically linked to the existence of homoclinic snaking in an infinite slot because their problem satisfied the conditions normally associated with the presence of this dynamics,⁴ namely, a subcritical primary bifurcation of a center-symmetric system of equations, and the existence of bistability between the conduction state and large-amplitude periodic states. Some Hopf bifurcations (HBs) were detected; however, the periodic orbits (POs) were not explored.

In the second paper, they found two subcritical secondary branches of localized states (L1 and L2), which bifurcate from a first subcritical branch of spatially periodic solutions (P1) in a codimension-two bifurcation. The secondary branches snake back and forth, before finishing in a “second” primary branch (P2) (other branches arise between P1 and P2), which bifurcates as the first from the conduction state. The flow at the first turning point of branch L1 consists of two clockwise vortices localized at the end of the cavity. New vortices are added near those already existent as one passes, following the curve, by the successive double-folds until the cavity is full, and the snaking branch connects with P2. In contrast, the flow at the first turning point of branch L2 forms a single vortex in the center of the container. Two vortices above and below the initial are added at each pass through a double-fold of L2, filling the domain. Finally, it rejoins P2 and L1. Solutions on L1 and L2 lose and regain stability as the branches snake toward larger amplitudes. The dynamics of the snaking branches of solutions in three-dimensional containers of square horizontal section were subsequently studied.¹ In this case, two branches keeping all the symmetries bifurcate at once from the conduction state, and undergo homoclinic snaking. Secondary symmetry-breaking bifurcations on the primary snaking branches generate secondary snaking branches.

Center-symmetric steady double diffusive convection in slots with periodic vertical boundary conditions in the BBC were also studied recently.¹³ The authors explored the changes of the snaking branches and the structure of the center-symmetric localized flow when the primary bifurcation evolved from subcritical to supercritical and the bistability was lost. The parameter Pr was reduced from $Pr = 1$ to $Pr < 0.06$, past the codimension-two point, for a mixture of $Le^{-1} = 5$. The increased complexity of the bifurcation diagrams at low Pr was attributed to the transition from buoyancy-driven rolls at high Pr to those driven by inertia and buoyancy forces.

The invariance of the solutions by vertical translations disappears in bounded rectangular cavities. Xin *et al.* showed that the type of primary bifurcation depends on Γ . The curves of TBs and PBs

decrease asymptotically to a constant value as $\Gamma \rightarrow \infty$, crossing each other. The transition is a TB for small Γ up to 2.14, it is a PB up to 3.4 where it changes again.¹⁵ It will be seen here that for $\Gamma = 8$ it is a PB.

Localized states of double diffusive convection in finite rectangular slots maintaining the center symmetry were studied¹⁴ to understand how departures from the BBC affect these patterns. In this case, a conduction state and Ra_c^T does not exist, and a global steady circulation develops as soon as the external gradients are established. It was found that the localized vortices near the horizontal sides develop from this large-scale flow. They persist over a wider range of buoyancy ratios, $\tilde{N}$, than those located inside, provided $\tilde{N} > -1$. Moreover, they disappear as the snaking of their solution branch ceases.

Different experimental setups have been used to maintain constant temperature and concentration at both lateral boundaries.^{6,8} In real situations, aside the finiteness of the domains, Soret and Dufour effects are present. A bifurcation diagram near the BBC for a H_2 -Xe gas mixture of $Pr = 0.168$ including both effects was computed for $\Gamma = 8$ in Sánchez and Net.¹¹ It was shown that only a few traces of the snaking remain, but extended, center-localized, and wall-attached patterns of steady convection with and without center symmetry subsist. In the first case due to the evolution of the flow along the primary branch, and in the second because this branch loses stability in PBs, which break the symmetry of the primary solution. It was also shown that the time periodic dynamics, due to the shear of the steady field, is delayed up to very high Ra^T .

In this paper, we are interested in studying the influence of the Soret effect and the symmetry breaking on the dynamics of the steady and stable periodic flows in finite slots for the BBC. The equations and their numerical treatment are discussed in Sec. II, including the variational equations to compute the orbits by continuation methods, and their stability with respect to any type of perturbation. In Sec. III, the influence of the Soret effect on the onset of convection is discussed. In Secs. IV and V, a detailed bifurcation diagram, including primary and secondary branches of steady states and POs, up to large Ra^T is analyzed. The paper concludes with the analysis of some relevant bifurcations detected on the branches of steady solutions in Sec. VI, and with a summary and discussion of the results in Sec. VII.

II. MATHEMATICAL FORMULATION

The dynamics of a binary mixture in a rectangular cavity, Ω , of width d , height h , and aspect ratio $\Gamma = h/d$ is numerically studied using a model previously derived,^{11,12} but omitting the Dufour effect. The lateral boundaries are taken perfectly conducting and permeable, and the top and bottom insulating and impermeable. All sides are supposed non-slip, namely, the velocity field is $\mathbf{v} = (u, v) = (0, 0)$ there. The fluid is subject to a constant vertical gravity, $\mathbf{g} = -g\mathbf{j}$, $\mathbf{j}$ being the unit vector pointing upward.

The mass, momentum and energy equations are written in the Boussinesq approximation with a linear density dependence on the dimensional temperature, T^* , and concentration of the denser component, C^* , as

$$\rho^*(T^*, C^*) = \bar{\rho}^*(1 - \alpha(T^* - \bar{T}^*) + \beta(C^* - \bar{C}^*)), \quad (1)$$

with $\alpha > 0$ and $\beta > 0$ being the thermal and the solutal expansion coefficients, respectively, measured at the average temperature $\bar{T}^*$ and concentration $\bar{C}^*$ at which the mean density is $\bar{\rho}^*$. In addition, they

are adimensionalized by taking d , ΔT^* and d^2/κ as longitude, temperature and time scales, and κ being the thermal diffusivity. The concentration is rescaled with ΔC^* . The system is described in terms of the streamfunction, ψ , by the dimensionless equations:

$$\partial_t \Delta \psi + J(\Delta \psi, \psi) = \text{Pr} \Delta^2 \psi + \text{Ra}^T \text{Pr} (1 - \partial_x \Theta) - \text{Ra}^C \text{Le} (1 - \partial_x \Sigma), \quad (2)$$

$$\partial_t \Theta + J(\Theta, \psi) = \Delta \Theta + \partial_y \psi, \quad (3)$$

$$\partial_t \Sigma + J(\Sigma, \psi) = \text{Le} \Delta \Sigma + \text{Pr} \text{Se} \text{N} \Delta \Theta + \partial_y \psi. \quad (4)$$

where $J(f, h) = \partial_x f \partial_y h - \partial_y f \partial_x h$, and $\mathbf{v} = (\partial_y \psi, -\partial_x \psi)$. Moreover Θ refers to the perturbation of the temperature with respect to a linear state, and Σ is the perturbation of the concentration with the same lifting.¹¹

The five physical parameters are the thermal Rayleigh, Ra^T , the solutal Rayleigh, Ra^C , the Prandtl, Pr , and the Lewis, Le , numbers, and the separation ratio, Se , defined as

$$\text{Ra}^T = \frac{\alpha g \Delta T^* d^3}{\nu \kappa}, \quad \text{Ra}^C = \frac{\beta g \Delta C^* d^3}{D \kappa}, \quad \text{Pr} = \frac{\nu}{\kappa}, \quad (5)$$

$$\text{Le} = \frac{D}{\kappa}, \quad \text{Se} = \frac{\beta K_T}{\alpha T^*}.$$

In the definitions, ν accounts for the kinematic viscosity, D for the mass diffusion coefficient, and K_T for the thermal diffusion ratio. Moreover, $\text{N} = \text{Ra}^T / \text{Ra}^C$ is the Rayleigh number ratio.

The boundary conditions for u and v translate into

$$\psi = \partial_n \psi = 0 \text{ on } \partial \Omega, \quad (6)$$

where n indicates the direction normal to the boundary. Those for Θ and Σ are

$$\Theta = 0 \quad \text{on } x = 0, 1 \quad \text{and} \quad \partial_y \Theta = 0 \quad \text{on } y = 0, \Gamma, \quad (7)$$

$$\Sigma = 0 \quad \text{on } x = 0, 1 \quad \text{and} \quad \partial_y \Sigma = 0 \quad \text{on } y = 0, \Gamma. \quad (8)$$

Equations (2)–(4) together with boundary conditions (6)–(8) are $\mathcal{L}_2$ equivariant, i.e., they remain invariant under the center symmetry.

$$\mathcal{S} : (t, x, y, \psi, \Theta, \Sigma) \rightarrow (t, 1 - x, \Gamma - y, \psi, -\Theta, -\Sigma). \quad (9)$$

The spatial discretization of the equations and the time integration schemes were already described in detail recently.^{10–12} A study of the dependence of several global parameters on the size of the mesh was included to establish that required to have accurate results. Let just say, that the functions ψ , Θ , and Σ were approximated by a pseudo-spectral collocation method on a mesh of $n_x \times n_y = 40 \times 150$ Gauss–Lobatto points, and that semi-implicit (IMEX) high-order time integration methods were used.

The computation of stationary states and POs by continuation methods and their stability requires solving the first variational equations of system (2)–(4), which depend on the parameter used in the continuation. The methods and the advantages of their use were also described formerly.^{10,12} We are interested in the BBC in this study, then $\text{Pr} \text{Ra}^T = \text{Ra}^C \text{Le}$, and the variational equations for the perturbations $(\tilde{\psi}, \tilde{\Theta}, \tilde{\Sigma})$, and δRa^T reduce to

$$\begin{aligned} \partial_t \Delta \tilde{\psi} + J(\Delta \tilde{\psi}, \psi) + J(\Delta \psi, \tilde{\psi}) &= \text{Pr} \Delta^2 \tilde{\psi} - \text{Ra}^T \text{Pr} (\partial_x \tilde{\Theta} - \partial_x \tilde{\Sigma}) \\ &+ \delta \text{Ra}^T \text{Pr} (\partial_x \Sigma - \partial_x \Theta), \end{aligned} \quad (10)$$

$$\partial_t \tilde{\Theta} + J(\tilde{\Theta}, \psi) + J(\Theta, \tilde{\psi}) = \Delta \tilde{\Theta} + \partial_y \tilde{\psi}, \quad (11)$$

$$\partial_t \tilde{\Sigma} + J(\tilde{\Sigma}, \psi) + J(\Sigma, \tilde{\psi}) = \text{Le} \Delta \tilde{\Sigma} + \text{NPrSe} \Delta \tilde{\Theta} + \partial_y \tilde{\psi}, \quad (12)$$

with boundary conditions,

$$\tilde{\psi} = \partial_n \tilde{\psi} = 0 \quad \text{on } \partial \Omega. \quad (13)$$

$$\tilde{\Theta} = 0 \quad \text{on } x = 0, 1 \quad \text{and} \quad \partial_y \tilde{\Theta} = 0 \quad \text{on } y = 0, \Gamma, \quad (14)$$

$$\tilde{\Sigma} = 0 \quad \text{on } x = 0, 1 \quad \text{and} \quad \partial_y \tilde{\Sigma} = 0 \quad \text{on } y = 0, \Gamma. \quad (15)$$

Notice that $\text{N} = \text{Le}/\text{Pr}$ positive refers to enforcing initial opposing buoyancy effects, i.e., it is assumed that the variation of the density due to the temperature gradient is compensated by that due to the concentration gradient. Its value and sign differs from the coefficient $\tilde{N} = -1$ defined by other authors² because α and β are taken positive here, and the scaling of the equations is different.

III. INFLUENCE OF THE SORLET TERM ON THE STABILITY OF THE CONDUCTION STATE

An Ar–CO₂ gaseous mixture of $\text{Pr} = 0.683$ and $\text{Le} = 1.085$ in a rectangular cavity of aspect ratio $\Gamma = 8$ has been chosen as example to evaluate the influence that the Soret effect can have on the onset of convection, and on the primary and secondary flows. The same parameters and aspect ratio were used previously for a mixture contained between impermeable vertical sides.¹⁰ In that case, there was no conduction state, and the former stable steady flows develop a global circulation that becomes a three-vortex structure at very low Ra^T when the fluid is a binary mixture of low Pr . These steady solutions satisfied the center symmetry,

$$\psi(1 - x, \Gamma - y) = \psi(x, y), \quad (16)$$

$$\Theta(1 - x, \Gamma - y) = -\Theta(x, y), \quad (17)$$

$$\Sigma(1 - x, \Gamma - y) = -\Sigma(x, y), \quad (18)$$

and the velocity field fulfills

$$u(1 - x, \Gamma - y) = -u(x, y), \quad (19)$$

$$v(1 - x, \Gamma - y) = -v(x, y), \quad (20)$$

i.e., they were invariant by $\mathcal{S}$.

The coefficient of the Soret term for Se as low as -0.08 is $\text{NPrSe} = -0.0868$, more than one order of magnitude lower than the other coefficients of Eq. (4). However, by omitting the Soret effect ($\text{Se} = 0$), the primary instability takes place at $\text{Ra}_c^T = 84\,665.99$ instead of at $\text{Ra}_c^T = 41\,889.51$ with $\text{Se} = 0.08$ or at $\text{Ra}_c^T = 3\,998\,116$ with $\text{Se} = -0.08$; so a weak negative Soret effect has a strong stabilizing influence, and a weak positive Soret effect has a strong destabilizing one (see below).

The critical value $\text{Ra}_c^T = 84\,665.99$ ($\Gamma = 8$) gives $\text{Ra}_c^T |\text{Le}^{-1} - 1| = 6632.82$, therefore it matches very well the critical relation $\text{Ra}_c^T |\text{Le}^{-1} - 1| = 6688$ when $\Gamma = 7$, found by Ghorayeb and Mojtabi,⁵ and the asymptotic convergence mentioned in Sec. I as $\Gamma \rightarrow \infty$.

The primary instability is a PB when $\Gamma = 8$ and $\text{Se} = -0.08$, so the critical real eigenfunctions break the symmetry $\mathcal{S}$ of the equations, but they satisfy the center antisymmetry:

$$\psi(1 - x, \Gamma - y) = -\psi(x, y), \quad (21)$$

$$\Theta(1-x, \Gamma-y) = \Theta(x, y), \quad (22)$$

$$\Sigma(1-x, \Gamma-y) = \Sigma(x, y). \quad (23)$$

Moreover, the critical eigenfunctions $(\psi_i, \Theta_i, \Sigma_i)$, $i = 1, 2$, of real eigenvalue $\lambda = 0$, corresponding to $\pm Se$, respectively, are related by a transform consistent in a reflection symmetry with a multiplicative factor for Σ , which depends on the parameters.

$$\psi_1(1-x, y) = \psi_2(x, y), \quad (24)$$

$$\Theta_1(1-x, y) = \Theta_2(x, y), \quad (25)$$

$$\Sigma_1(1-x, y) = C_\Sigma \Sigma_2(x, y). \quad (26)$$

Since they are eigenfunctions, a real global factor could be added multiplying one of the two sets. It has been taken as +1 here and in the following discussion. Figure 1 shows a sample of the contour plots of the related eigenfunctions $(\psi_1, \Theta_1, \Sigma_1)$ and $(\psi_2, \Theta_2, \Sigma_2)$ at left and right, respectively.

The above relations hold because the linear equations for the eigenfunctions of null eigenvalue have a special structure for any set of parameters (Pr , Le , Se , Ra^C , and Ra^T). Equations (2)–(4) linearized about the conduction state, $(\psi, \Theta, \Sigma) = 0$, and with $\lambda = 0$ can be written as

$$0 = Pr\Delta^2\psi - Ra^C Le(\partial_x\Theta - \partial_x\Sigma), \quad (27)$$

$$0 = \Delta\Theta + \partial_y\psi, \quad (28)$$

$$0 = Le\Delta\Sigma + (LeSe - 1)\Delta\Theta, \quad (29)$$

when the BBC relations $Ra^T Pr = Ra^C Le$ and $NPr = Le$ are taken into account, and Eq. (28) is used to substitute the term $\partial_y\psi$ in Eq. (12).

Equation (29), together with the boundary conditions (14) and (15), imply the proportionality

$$\Sigma = \frac{1 - LeSe}{Le} \Theta. \quad (30)$$

Then, by substituting Σ from Eq. (30) into Eq. (27), the eigenvalue problem reduces to

$$0 = Pr\Delta^2\psi - Ra^C[Le(1+Se) - 1]\partial_x\Theta, \quad (31)$$

$$0 = \Delta\Theta + \partial_y\psi. \quad (32)$$

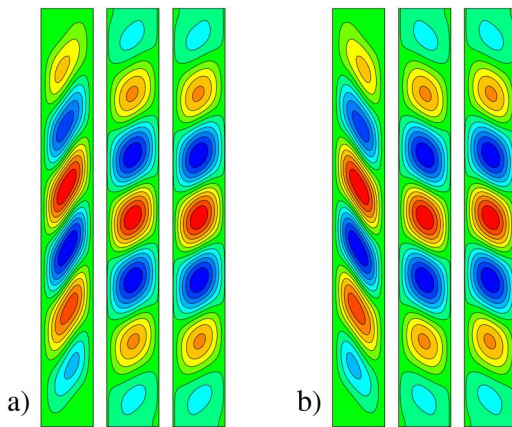


FIG. 1. Contour plots of the eigenfunctions for $Le = 1$. (a) From left to right $(\psi_1, \Theta_1, \Sigma_1)$ with $Se = 0.2$ and (b) $(\psi_2, \Theta_2, \Sigma_2)$ with $Se = -0.2$. The other parameters are $Pr = 0.683$, $N = 1/Pr$, and $\Gamma = 8$. The intensity of the color is local in each plot.

Equation (31) helps to understand the deep effect of the Soret term in the onset of convection in the BBC when the mass and thermal diffusivities are similar ($Le \approx 1$). For instance, the ratio of $|Le(1+Se) - 1|$ for $Se = 0.08$ and $Se = -0.08$ is 95.44 when $Le = 1.085$. Therefore, Ra^C and also Ra^T must vary in this factor when Pr is constant. This explains the jump of critical values when $Le \approx 1$, as in the preceding example. On the other hand, the tilting of the contour plots of the eigenfunctions of Fig. 1 can be understood from Eq. (1) written for the perturbations Θ and Σ in the BBC, i.e., when $\alpha\Delta T^* = \beta\Delta C^*$. By using Eq. (30), the non-dimensional density reduces to

$$\rho = \rho_{eq} + \alpha\Delta T^*(a-1)\Theta, \quad (33)$$

with $a = (1 - LeSe)/Le$. The value of a determines the increase or decrease in ρ with Θ with respect to the homogeneous equilibrium state. When $a > 1$, i.e., for $Se < (1/Le) - 1$, ρ increases in the points where Θ increases, and vice versa. When $Se > (1/Le) - 1$, ρ decreases in the points where Θ increases. When $Le = 1$ as in Fig. 1, ρ increases with Θ if $Se < 0$, and decreases when $Se > 0$. Consider, in the latter case, a fluid particle slightly displaced from the left wall toward the right, close to the vertical middle of the domain, at the threshold of convection. Since $\Theta > 0$ there [see Fig. 1(a)], its density decreases, the fluid particle rises, and the streamfunction contours incline upward to the right. When $Se < 0$, the effect is the opposite as in Fig. 1(b). In general, when $Le \neq 1$, the sign of the combined parameter $1 - Le(1+Se)$ determines the inclination of the convective cells.

If the ratio of parameters

$$\frac{Ra^C[Le(1+Se) - 1]}{Pr}, \quad (34)$$

does not change, neither does the eigenvalue problem, and the critical positive Rayleigh numbers corresponding to the two sets of parameters (Pr_i, Le_i, Se_i), $i = 1, 2$, satisfy

$$\frac{Ra_{1c}^C}{Ra_{2c}^C} = \left| \frac{Pr_1[Le_2(1+Se_2) - 1]}{Pr_2[Le_1(1+Se_1) - 1]} \right|. \quad (35)$$

If the ratio inside the absolute value is positive, then $\psi_1(x, y) = \psi_2(x, y)$, $\Theta_1(x, y) = \Theta_2(x, y)$, and $\Sigma_1(x, y) = C_\Sigma \Sigma_2(x, y)$, with

$$C_\Sigma = \frac{Le_2(Le_1Se_1 - 1)}{Le_1(Le_2Se_2 - 1)}, \quad (36)$$

according to Eq. (30). If it is negative, the eigenfunctions are related as in Eqs. (24)–(26). This introduces a change of sign in the term $\partial_x\Theta$ of Eq. (31), which compensates that of $Le(1+Se) - 1$, and does not modify that of the Laplacians. In the previous example, changing the sign of Se , while keeping Pr and Le fixed, gives rise to a negative ratio.

Notice that the inclusion of the Soret term for equal thermal and solutal diffusivities ($Le_{1,2} = 1$) implies that the critical Ra^T at which the conduction state destabilizes is the same for $\pm Se$ if $Pr_1 = Pr_2$, according to Eq. (35). For instance, for $Se = \pm 0.2$ and $Pr_{1,2} = 0.683$, $Ra_c^T = 33\,164.10$. The sign of Θ and Σ is the same in each eigenfunction because in both cases $1 - LeSe > 0$, according to Eq. (30).

When the Soret term is neglected and $Le = 1$, there is no instability since the momentum equation (31) becomes a diffusive equation.⁵ When $Se \neq 0$ and $Le \neq 1$, the critical parameter changes from

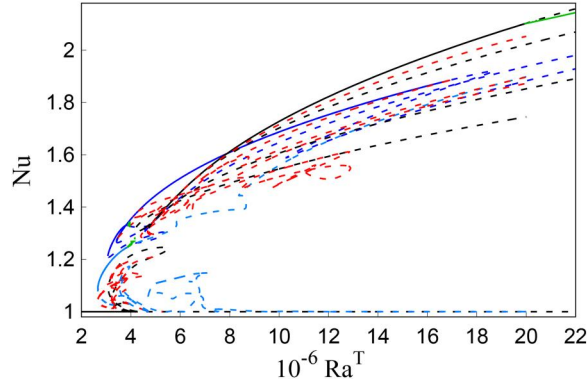


FIG. 2. Bifurcation diagram showing the branches of solutions for $\Gamma = 8$, $Pr = 0.683$, $Le = 1.085$, and $Se = -0.08$.

$Ra_1^T(1 - Le^{-1})$ to $Ra_2^T(1 - Le^{-1} + Se)$, i.e., there is a relative shift $1 + [LeSe/(Le - 1)]$ in the critical Ra^T , denoting that the stronger influence takes place when the solutal and thermal diffusivities are similar $Le \approx 1$ (gaseous mixtures), and that for $Le \ll 1$ (liquid mixtures), it is negligible if $|Se|$ is small.

IV. ANALYSIS OF THE BIFURCATION DIAGRAM: STEADY SOLUTIONS

The bifurcation diagram of Fig. 2 corresponds to the Ar-CO₂ mixture previously mentioned with $Se = -0.08$. Their physical parameters give a constant $N = 1.588\,579\,8$ in the BBC, the continuation parameter selected is Ra^T , and $Ra^C = Ra^T/N$.

Some global quantities were computed to monitoring the solutions; the kinetic energy density, K , and the Nusselt, $Nu = \partial_x T$, and Sherwood, $Sh = \partial_x C$, numbers, the latter two averaged along the wall $x = 1$. In the case of POs they were also averaged over a period. However, an amplitude, A , related to the expansion of the streamfunction in Chebyshev polynomials has been used in the detailed figures of the bifurcation diagram since the mean values of the branches of low Nu are often very close to each other.

Let $\psi_{ij} = \psi(x_i, y_j)$ be the values of ψ at the collocation nodes (x_i, y_j) . The index i is set to $i_0 = n_x/2$, and the values $\psi_{i_0, j}$, $j = 0, \dots, n_y + 1$ are transformed to obtain the approximation:

$$\psi(x_{i_0}, y) = \sum_{k=0}^{n_y+1} \alpha_k T_k(2y/\Gamma - 1), \quad (37)$$

where $T_k(z)$, $z \in [-1, 1]$ are the Chebyshev polynomials of the first kind, and the coefficients α_k satisfy the linear system,

$$\psi(x_{i_0}, y_j) = \sum_{k=0}^{n_y+1} \alpha_k T_k(2y_j/\Gamma - 1), \quad j = 0, \dots, n_y + 1. \quad (38)$$

Then the amplitude is defined as

$$A = \sum_{k=0}^{n_y+1} \alpha_k / 2^k. \quad (39)$$

Its representation separates the lower branches of solutions better than the previous magnitudes.

The diagram will be described in detail by parts because of its complexity. The intersections of branches marked with dots correspond to PBs and TBs, those marked with triangles refer to HBs, and those marked with squares to period doubling bifurcations of periodic orbits (PDBs). Solid and dashed lines mean stable and unstable solutions, respectively. Black and blue lines indicate symmetric branches of steady solutions, those in red, orange, and magenta curves of SB steady solutions, and those in green denote POs. The crosses of the color of the corresponding branch indicate the points where the contour plots are shown. Only one of the two symmetry-conjugated branches arising at PBs is included in each case. These criteria will apply along the text.

Figure 3 displays details of the bifurcation diagram along the conduction state, $(\psi, \theta, \Sigma) = 0$, represented by the horizontal black line (B0). It is stable up to a PB at $Ra^T = 3.9981 \times 10^6$, where a subcritical branch of SB solutions starts [branch B1 in Fig. 3(a)]. There is a TB very close, at $Ra^T = 4.0056 \times 10^6$, linked to the branch, B2, of symmetry-preserving (SP) solutions. This scenario is repeated several times in the range of the Ra^T explored. Four examples of consecutive pairs of PB-TB or TB-PB are shown. The first pair in Fig. 3(a) is as mentioned earlier. Each branch of SB solutions starts at B0 and ends at a nearby SP branch B2 or B9 (in black and blue colors, respectively, in Figs. 3 and 4). All the solutions, in these diagrams, are unstable except B0 up to the first PB. The contour plots ψ and Θ of these flows at any non-trivial branch show extended patterns of convection of six or more vortices, whose number increases and become more irregular with Ra^T (see optionally Fig. SM3-1 of the supplementary material). It must be clarified that the numbering of the branches has no special meaning, it just has to do with the order in which they were computed.

Branches B2 and B9 are the only curves of SP flows that intersect B0 at the TBs of Fig. 3, as shown in Fig. 4. The first, B2, forms a closed curve and contains the first two TBs on B0. The second, B9, extends to large values of Ra^T and contains several of them.

The curve B2 has eight turning points [see Figs. 3(a) and 4]. Starting from the saddle-node bifurcation (SNB) located at $Ra^T = 3.2306 \times 10^6$ where the solutions stabilize, and moving to high amplitude, A , the number of cells changes from two to four

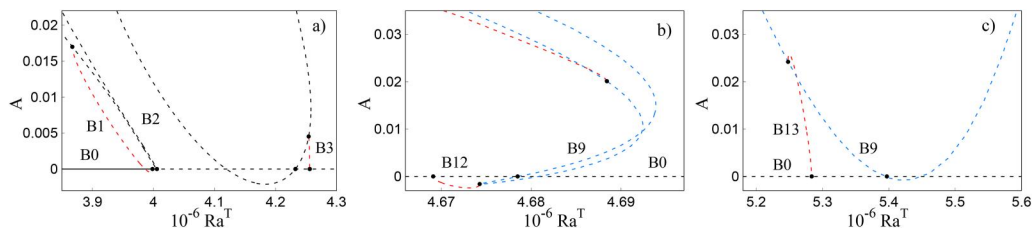


FIG. 3. Detail of the first eight bifurcations of the conduction state and (a) of the emerging branches B1, B2, and B3, (b) B12 and B9, and (c) B13 and B9.

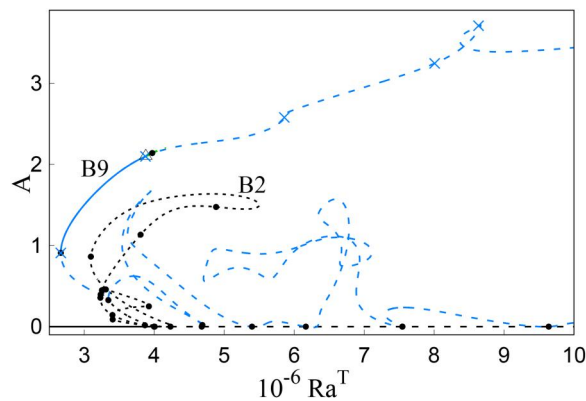


FIG. 4. Detail of the branches B2 and B9 of center-symmetric steady solutions, bifurcated from the conduction state.

when passing the rightmost turning point. Then, the curve reaches a TB at $Ra^T = 4.2325 \times 10^6$ after the leftmost one. Following the curve, the solutions evolve to a three-cell pattern, which transforms into a single cell localized at the center of the box. The other TB is located at $Ra^T = 4.0056 \times 10^6$ before closing the curve. The segment of stable solutions on B2 is very short, bounded by the SNB at $Ra^T = 3.2306 \times 10^6$ and a PB at $Ra^T = 3.2340 \times 10^6$ [see Fig. 6(a) on the left]. The change of sign of ψ at the TBs and the evolution of the patterns of convection along B2 can optionally be seen in Fig. SM5-1 of the [supplementary material](#).

Branch B9 will be described in two parts. The upper and lower include the solutions above and below the leftmost SNB located at $Ra^T = 2.6644 \times 10^6$, respectively (see Fig. 4). Contour plots of ψ in the upper part of B9 can be seen in Fig. 5, and those of the lower part in Fig. SM5-2 in the [supplementary material](#) if necessary. In this type

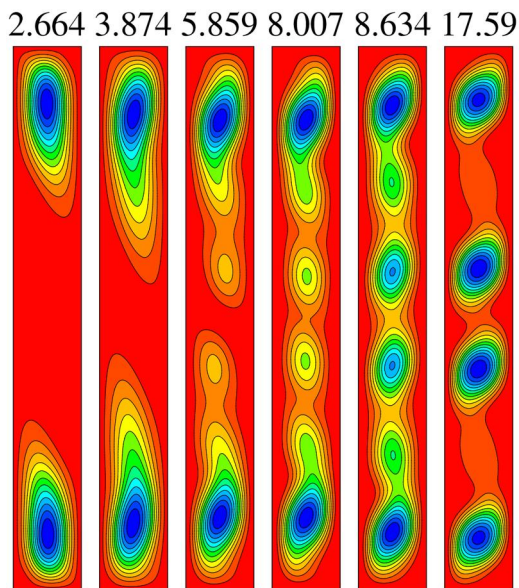


FIG. 5. Contour plots of ψ for steady states along the upper part of branch B9. The number above the plots is $Ra^T \times 10^{-6}$.

of figures the number above each snapshot is $Ra^T \times 10^{-6}$. The perturbation of the concentration, Σ , is not included because the contour plots are very similar to those of Θ in the BBC, although they are no longer proportional like at onset. Along the upper branch, the solutions transform from two cells localized at the endpoints of the box to four eye-shaped vortices. Two of them remain attached to the endpoints and the other two grow in the center. This branch is stable from the SNB up to a HB at $Ra^T = 3.8787 \times 10^6$ (see Sec. V). The lower part crosses B0 in TBs at $Ra^T = 4.6785 \times 10^6$, 5.3972×10^6 , 6.1661×10^6 , 7.5461×10^6 , 9.6385×10^6 , and 12.289×10^6 . Many different patterns of convection are found along the lower part of B9, with the number of cells changing mainly at the SNBs and TBs, among them localized states with two or one small cell, as well as single elongated vortices filling a large fraction of the slot. Solutions with a large number of cells appear at the higher Ra^T after several TBs. They are of low amplitude and very unstable.

Figures 6(a) and 6(b) show details of branches B2 and B9, and those of SB steady solutions B5, B6 (red), B7, B8 (magenta), and B10, B11 (orange) starting and finishing on them, always at PBs. Curves B5 and B6 start on B2 and B9, and finish on B9 and B2, respectively. Both are always unstable. Contour plots along them can be seen in Figs. 7(a) and 7(b). Branch B5 starts at $Ra^T = 3.2524 \times 10^6$ and ends at $Ra^T = 4.6884 \times 10^6$. The patterns suffer a drastic evolution from three to seven vortices. Branch B6 starts so close to the leftmost turning point of B9 commented above that slightly moving a second parameter it should be possible to capture the bicritical point. Their contour plots show the transition from two localized vortices near the lids to a single cell localized at the center. The first and last patterns of double-diffusive convection of Fig. 7(b) were previously found,⁵ also for the balanced case but for different values of Pr , Le , and Γ .

Branches B7 and B8 have their endpoints on branch B2 unlike B5 and B6. The curve B7 has four folds, it is always unstable and evolves from three to two vortices, similar to what happens following B8 [see Fig. 7(c)], but with one additional cell. The weak top vortex disappears on B7 while the bottom one reinforces (and symmetrically in the conjugated branch). The curve B8 starts being stable, branching from a pattern of two vortices at $Ra^T = 3.2340 \times 10^6$ in the short stable segment of branch B2 [first plot of Fig. 7(c)], and loses stability in a SNB at $Ra^T = 3.2375 \times 10^6$ (before the second plot). In the interval between these two solutions B8 undergoes a tiny loop, with a SNB located at $Ra^T = 3.2303 \times 10^6$. The interval of stability is so narrow that it would be very difficult to detect these non-symmetric stable solutions by direct simulation. The branch ends at $Ra^T = 3.9237 \times 10^6$, with a centered elongated cell, close to a SNB of branch B2.

There are two more branches of SB solutions with endpoints on branch B2. They are labeled as B10 and B11 in Fig. 6. Both are unstable, and as before, the change of the patterns of convection takes place near the turning points of the curves. The evolution of these patterns along the branches is similar to those of B7 and B8, B10 passing from three vortices, of almost equal intensity, to two elongated vortices that almost fill the slot. In contrast, B11 evolves from four vortices of weak intensity near the top and bottom sides to two vortices.

Two new main branches of steady solutions, B4 and B14, disconnected from the conduction state were found in addition. They are shown in Fig. 8. Both preserve the center symmetry, and reach high mean values and amplitude, A , at large Ra^T . The lower, B4, has two

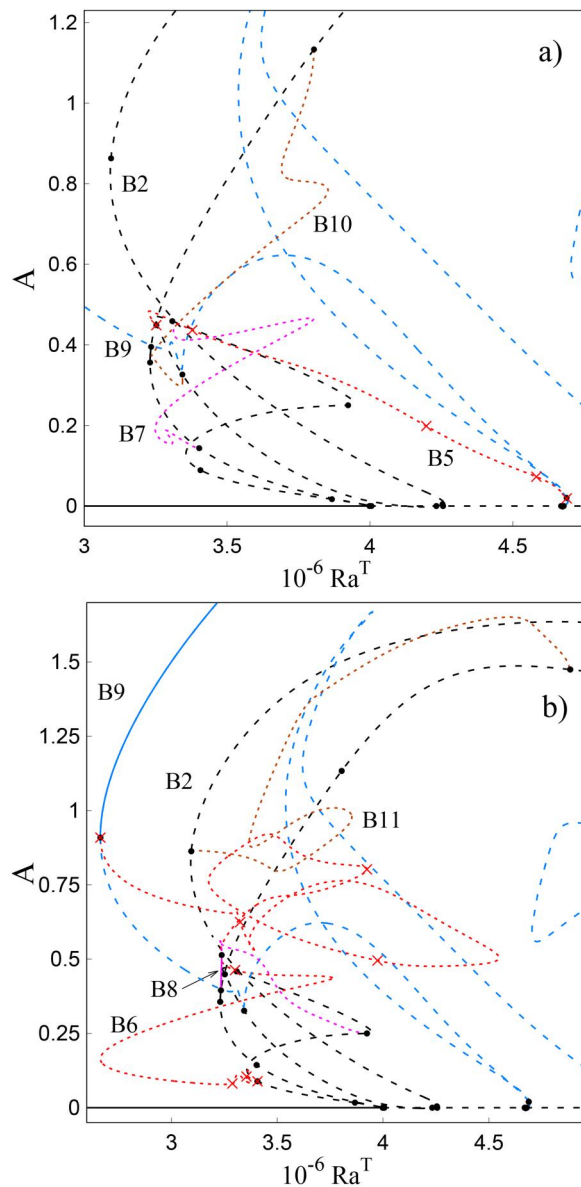


FIG. 6. Detail of the branches (a) B5, B7, and B10 and (b) B6, B8, and B11 of SB solutions connecting the symmetric branches B2 and B9 or an own branch after PBs.

stability intervals. At low Ra^T , and starting from $Ra^T = 3.0754 \times 10^6$, the curve is stable up to a HB at $Ra^T = 3.8899 \times 10^6$ (see Sec. V), then it undergoes two turning points and recovers stability from $Ra^T = 3.4784 \times 10^6$ up to $Ra^T = 16.9172 \times 10^6$, giving rise to a short bistability interval of steady solutions. It does not recover the stability after the two turning points located at high Ra^T . Moreover, there is also a small quadristability region of steady solutions before $Ra^T = 4 \times 10^6$ because B9 and the conduction state are also stable up to almost this value (see Fig. 2). The contour plots of ψ along the lower part of B4 are included in Fig. 9. They are plotted starting from the

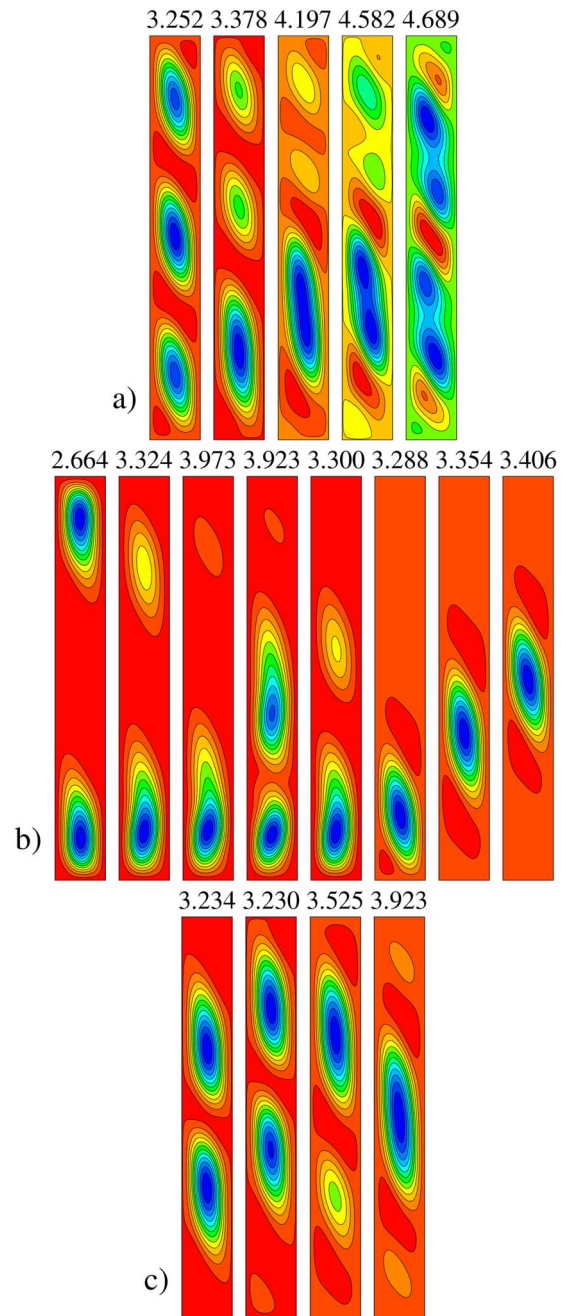


FIG. 7. Idem Fig. 5 along branches (a) B5, (b) B6, and (c) B8.

leftmost point at $Ra^T = 3.0754 \times 10^6$. The first and the last but one snapshots belong to stable solutions. The flow has three vortices in the short stable segment, and continues having four when the weak central vortex splits after the two initial folds. Stable extended flows of three and four vortices (Fig. 9) coexist with localized states (Fig. 5) at low Ra^T . The upper part of B4 is always unstable. In this case, the top and

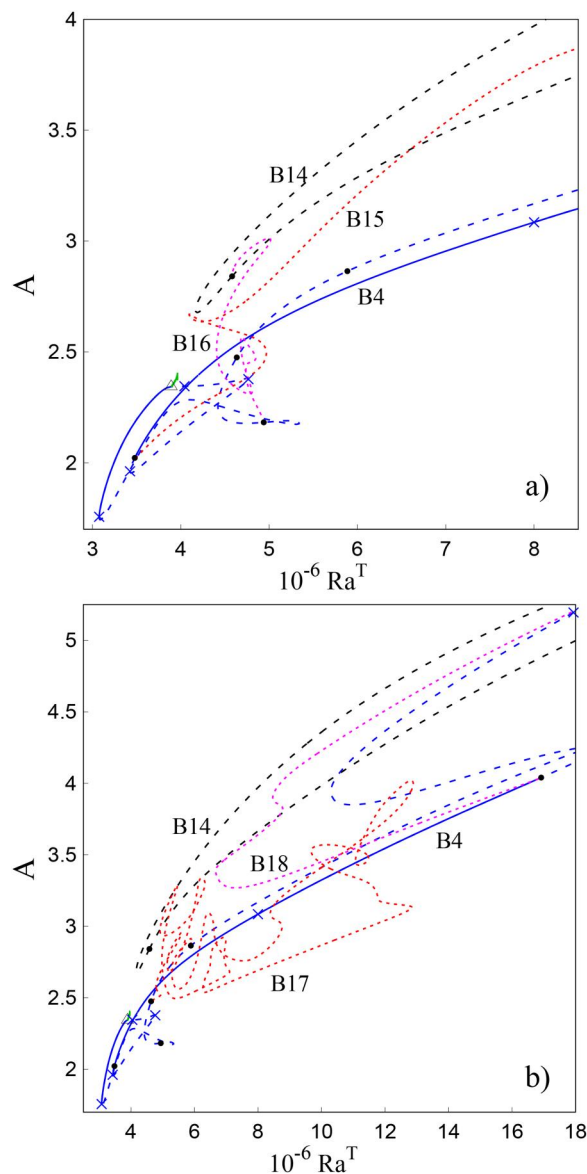


FIG. 8. Symmetric branches B4, B14, and B15, B16, B17, and B18 of asymmetric flows, arising at PBs of the former.

bottom cells (not shown) split at moderate Ra^T , generating a pattern of five vortices. The central one weakens at high Ra^T , and two pairs remain near the horizontal sides with an almost quiescent center.

Branch B14 has a single fold at $Ra^T = 4.1656 \times 10^6$, and is always unstable. The patterns around this fold have three vortices (not shown), the central one being very weak along both parts. The main difference between both segments is that the top and bottom vortices of the lower part of the curves split, similar to what happens in the upper part of B4.

All the branches bifurcating from B4 and B14 at PBs give rise to extended patterns of convection. Branch B15 arises at the PB where branch B4 recovers the stability. After a turning point and a small

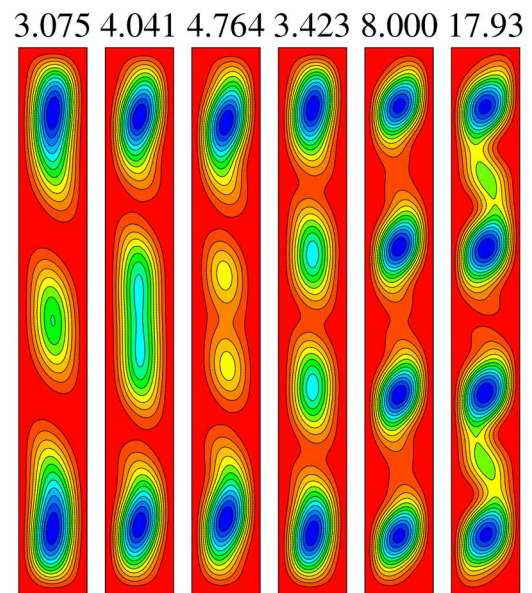


FIG. 9. Idem Fig. 5 along the lower branch B4.

loop, it extends up to very high parameter values. All the solutions computed are unstable, having at least five positive eigenvalues above $Ra^T = 7.2619 \times 10^6$. This secondary branch arises with four vortices that evolve to three by increasing the parameter, with the stronger vortex near the center of the box, unlike the solutions of the primary curve. Branch B16 is also unstable, connecting B4 and B14 after several turning points where the pattern of convection changes but maintains the five vortices of the bifurcation points, although the three central ones have a very weak intensity.

B17 is a SB branch which connects two points of B4. It reaches high Ra^T following an involved curve with several folds and loops. However, the two limiting PBs are placed at moderate values of the parameter. On this curve the initial pattern of convection has five vortices, and always maintains the upper and lower ones of high intensity like the solution at the bifurcation point. The important changes affect the three central vortices, mainly from the center to the bottom of the cavity, and to the top in the symmetry-conjugated branch. They move, weaken, and even disappear along the curve. B18 arises subcritically in a PB at $Ra^T = 16.9172 \times 10^6$, where B4 loses the stability. It attains moderate Ra^T , and after two folds, it continues to large Ra^T . It never becomes stable in the range studied. The flow always maintains the initial extended pattern of convection, with four strong initial vortices moving to different positions and having slight variations in the intensity.

A third primary branch, B19, disconnected from B0 was found at high amplitude, extending from low to high Ra^T (Fig. 10). The branch is symmetric, and turns at low Ra^T forming a small loop, and becoming stable in a PB at $Ra^T = 4.8529 \times 10^6$ up to $Ra^T = 19.9399 \times 10^6$, where it undergoes a HB (see Sec. V). In this case, the flow fills the box with five equispaced vortices in the stable part, and five regrouped in the unstable [see Figs. 11(a) and 11(b)]. The emerging SB branch, B20, remains always unstable. The patterns

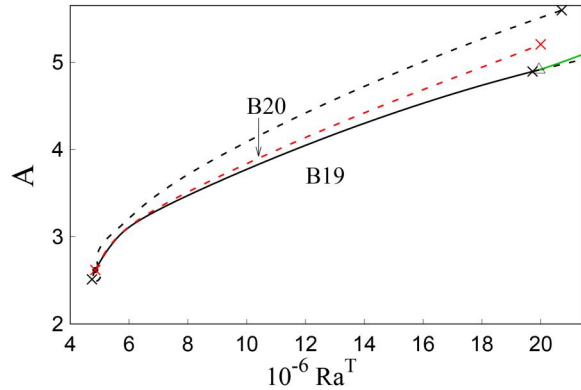


FIG. 10. Detail of branch B19 of SP and B20 of SB solutions.

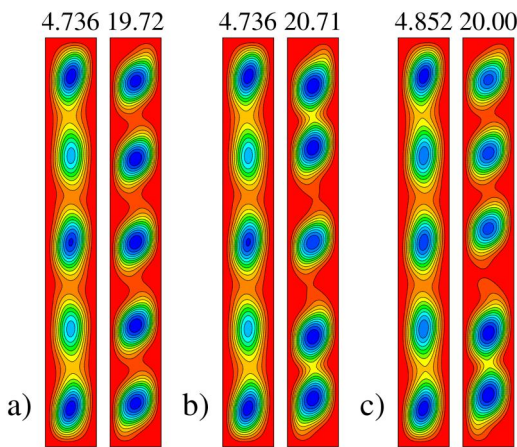


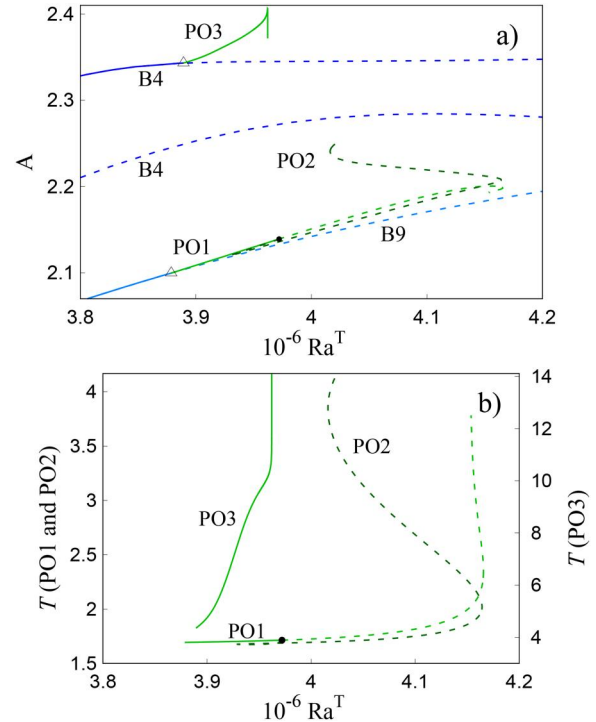
FIG. 11. Idem Fig. 5 along the (a) stable and (b) unstable parts of branch B19, and (c) along branch B20.

simply consist in the redistribution of the five vortices, forming groups of three and two in the upper and the lower parts of the cavity, respectively [see Fig. 11(c)].

V. PERIODIC ORBITS

Several HBs were found along the branches of steady solutions, but only the initially stable branches of POs and some branching from them were computed and included in green color in the bifurcation diagrams.

Figure 12(a) shows the branch of stable POs emerging when the localized patterns of B9 lose stability in a HB at $Ra^T = 3.8787 \times 10^6$. Branch PO1 is supercritical, and maintains the center symmetry of B9 (i.e., the POs are fixed cycles). However, it soon loses stability in a PB of periodic orbits (PBPO) at $Ra^T = 3.9737 \times 10^6$. By increasing Ra^T , a second real multiplier crosses the unit circle in a SNB of PO (SNPO) at $Ra^T = 4.1659 \times 10^6$. From this point, the leading eigenvalues start to increase, and the orbit does not recover the stability. The branch starts with period $T = 1.7143$, and at the SNPO it has increased until $T = 2.3533$. From this point it grows much faster, indicating that it approaches a homoclinic orbit [see Fig. 12(b)]. The oscillation consists


 FIG. 12. (a) Amplitude, A , and (b) period, T , vs Ra^T along branches PO1 and PO3 of POs, and branch PO2 arising at a subcritical pitchfork bifurcation of PO1.

in the lengthening and shortening of two elongated vortices attached to the horizontal sides toward the center of the slot that remains quiescent (Fig. 13, first row). The PBPO is subcritical and consequently the new branch PO2 is unstable, and continues to be so after each of its three turning points of Fig. 12. The fast increase in the period indicates that PO2 also finishes in a homoclinic bifurcation. The flow consists of the oscillation of the two different vortices attached at the top and bottom sides, as described above. The snapshots are similar to those of Fig. 13(a) but they are no longer center symmetric. However, the orbits retain a spatiotemporal symmetry. Applying the center-symmetry to the flow is equivalent to evolving the solution half a period (i.e., the POs are symmetric cycles).

Another HB takes places at $Ra^T = 3.8899 \times 10^6$ on branch B4 of extended patterns of convection (see also Fig. 12). The bifurcation is supercritical, and PO3 retains the center-symmetry. It remains stable up to its end when it undergoes a homoclinic bifurcation near $Ra^T = 3.9621 \times 10^6$. The oscillation consists in a slow lengthening and weakening of the central vortex followed by a fast shortening and strengthening. The top and bottom vortices lengthen in the opposite vertical direction to the central one, without losing their intensity (Fig. 13, second row).

A new transition to stable symmetric POs is found on B19 at $Ra^T = 19.9399 \times 10^6$. The emerging curve, PO4, is shown in Fig. 14. It is the longest branch of POs detected, and it becomes unstable giving rise to a Feigenbaum cascade of PDBs, depicted in different green tones. The subsequent PDBs computed are located at $Ra^T = 34.4344 \times 10^6$, 39.8413×10^6 , 41.4005×10^6 , 41.7309×10^6 , and 41.8070×10^6 . The period of PO4 is $T = 0.065974$ at the HB, very small compared to

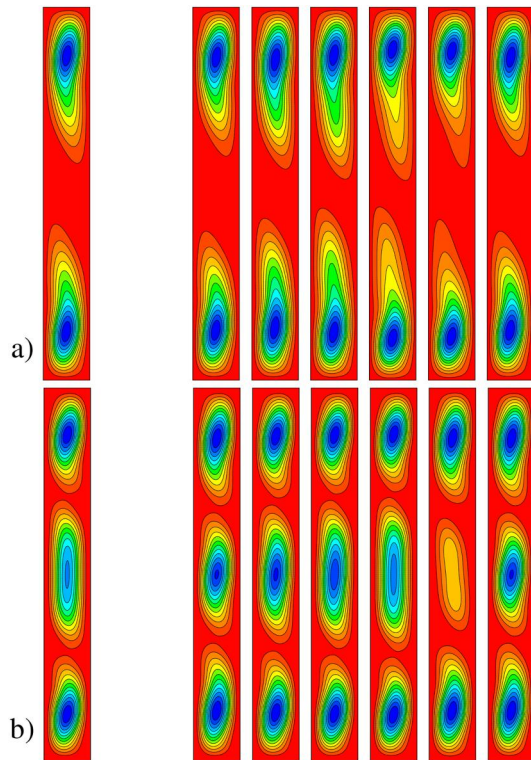


FIG. 13. Contour plots of ψ (a) of a stable steady flow at $Ra^T = 3.8787 \times 10^6$, on the left, and of the temporal evolution of a stable PO during a period $T = 1.7115$ at $Ra^T = 3.9641 \times 10^6$, on the right. The snapshots are taken in time intervals $\Delta t = T/5$, and (b) of the stable steady flow at $Ra^T = 3.8855 \times 10^6$, and the stable PO of $T = 9.5864$ at $Ra^T = 3.9503 \times 10^6$.

that of the preceding POs, and in addition, it decreases by increasing the parameter. The flow mainly consists of five equispaced clockwise vortices that maintain the position at any time (see Fig. 15), separated by a weak anticlockwise circulation. They change their shape from rhomboidal to ellipsoidal in a period, while a general anticlockwise circulation starts to develop, surrounding the ellipsoidal vortices. Finally, it breaks down and the vortices recover their initial shape. When this happens, there is an injection of hot fluid to the interior of the cavity. At each PDB, the aforementioned sequence occurs two, four, eight, etc., times per period.

VI. ENERGY BALANCE AT THE BIFURCATION POINTS

The mechanisms leading to the onset of steady and periodic regimes from equilibria states, can be established from the kinetic energy balance of the perturbed field, obtained from the momentum equation.^{7,10,11} For time independent flows, the spatial average of the kinetic energy equation, with non-slip boundary conditions, is just a balance among the rate of kinetic energy generated by the shear of the steady field, $\bar{K}_{sh}$, by the work done by the thermal buoyancy per time unit, $\bar{K}_{bT}$, by that done by the solutal buoyancy, $\bar{K}_{bC}$, and the rate of energy dissipated by viscosity, $\bar{K}_{ds}$, because the rate of change of the total kinetic energy, $\bar{K}$, of the perturbation is zero. The contribution of these terms to the transitions of the steady flows to new steady patterns or to periodic flows is given in Table I. The main feature is the large

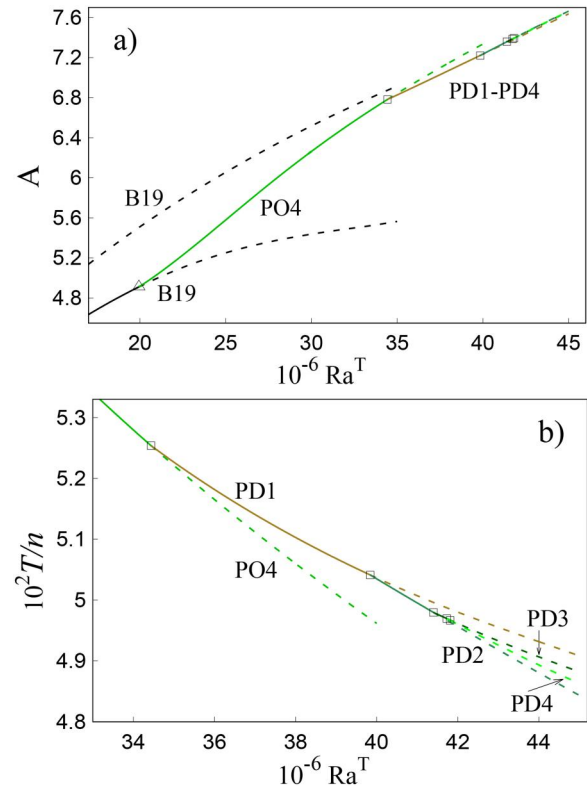


FIG. 14. (a) Amplitude, A , of the branch PO4 of POs, and branches arising at the subsequent period doubling bifurcations. (b) Periods T/n of these branches, with $n = 1, 2, 4, 8, \dots$

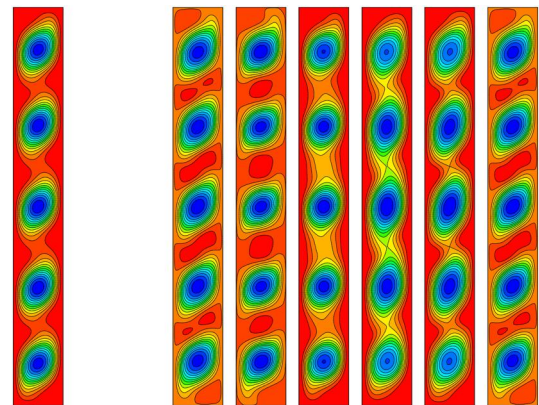


FIG. 15. Idem Fig. 13 for the stable steady flow at $Ra^T = 19.9399 \times 10^6$, and the stable PO of $T = 0.055440$ at $Ra^T = 30 \times 10^6$.

contribution of $\bar{K}_{bT}$ and $\bar{K}_{bC}$ to the power equilibrium, despite they almost balance each other. The rate of work $\bar{K}_{bC}$ usually contributes to the instability, while $\bar{K}_{bT}$ stabilizes the fluid. However, the balance at the HB of B19, giving rise to a period doubling cascade of POs behaves in the other way (see last line). Moreover, $\bar{K}_{ds}$ together with $\bar{K}_{sh}$ compensate the positive balance due to the buoyancy forces. Notice that $\bar{K}_{sh}$

tends to destabilize the fluid as usual, but it can even contribute to diminish $\bar{K}$, as happens in the PB of B2 and the SNB of B8 (fourth and seventh lines), where the branch B8 starts and changes stability. Moreover, $\bar{K}_{sh}$ is very small at low Ra^T except at the HBs where branches B9 and B4 lose stability (sixth and eleventh lines, respectively). The balance of $\bar{K}$ at the successive PBs and TBs points on B0 (first two lines) is qualitatively the same, with $\bar{K}_{bc}$ balanced by $\bar{K}_{bT}$ and $\bar{K}_{ds}$.

VII. SUMMARY AND DISCUSSION

Double-diffusive convection of an Ar–CO₂ mixture filling a rectangular slot has been studied. Lateral thermal and solutal gradients in the BBC were considered, with the Soret effect included in the model. The influence of this phenomenon on the onset of the primary instability has been discussed in detail. Analytical formulas to determine the relation between the critical parameters and eigenfunctions are given. Moreover, it is shown that the addition of the Soret term shifts the absence of linear instability of the thermosolutal problem for $Le = 1$ to $Le = 1/(1 + Se)$, and that the Soret effect will be important in mixtures with $Le(1 + Se) - 1 \approx 0$.

A detailed bifurcation diagram of the primary and secondary branches of steady flows and stable periodic orbits up to $10 Ra_c^T$ has been unfolded, and their stability analyzed. The large amount of turning points of the steady curves determines the coexistence of a multiplicity of extended patterns of convection at any Ra^T . Steady convection localized in the center of the cavity has been found on symmetric branches B2 and B9 at low amplitude, below and above the onset of convection from B0, respectively. The existence of center-symmetric localized steady states above onset is in agreement with previous studies,¹³ but here they arise from primary extended solutions bifurcating from B0. Stable convection attached to the top and bottom sides of the cavity, with a quiescent center, has been located on branch B9 at higher amplitudes, always below the critical parameter. These kinds of patterns were also found in the finite slot for the general case.^{3,11,14} In addition, three main symmetric branches, B4,

B14 and B19, not related to B0 have been found at high amplitude. B4 and B19 contain the largest segments of stable steady solutions computed, and B4, along with B9, give rise to multistability of extended and localized patterns of convection.

Localized SB patterns of one or two vortices, restricted to one side or near the center of the slot, were found on secondary branches connecting B2 and B9 or the own branches at low amplitude. For instance, the multiple turning points of B6, connecting B2 and B9, determine the coexistence of almost centered and wall-attached patterns even on the same branch, and B8 branching only from B2 arise with asymmetrical stable patterns of two elongated vortices at low amplitude. These types of convection survive when the BBC is weakly broken.¹¹ Among this variety of solutions, the stable steady patterns are those that tend to maximize the heat and mass transport, as can be seen in Fig. 2. Its upper envelope consists only of stable solutions up to the PDBs where the periodic dynamics starts to decrease Nu at large Ra^T . Therefore, the longest interval of steady and periodic flows is related to solutions not bifurcated from the conduction state.

The analysis of the power supplied by each term of the kinetic energy equation at the bifurcation points in the BBC with Soret effect shows that the rate of work done by both buoyancies is four or five orders of magnitude larger than the rate of generation of energy by the shear stresses, but the contribution of the two types of buoyancies is almost in balance. The thermal buoyancy is the main responsible for the onset of the oscillations in the convection at large amplitude. It contributes to stabilizing the fluid in any other case explored. Moreover, in the case at hand, and in contrast to preceding studies, the shear stresses can even help to dissipate energy (see fourth and eighth lines of Table I).

New patterns of stable extended and localized symmetric POs that maintain at any time the spatial structure of the steady states, from which they bifurcate, were found. They coexist with stable steady states of B4 at low Ra^T , and end just before the conduction state loses stability. These oscillations are generated mainly by $\bar{K}_{bc}$, and finish in homoclinic bifurcations, while those generated mainly

TABLE I. Power balance at some bifurcation points on the steady branches (BI, $l = 0, 2, 4, 8, 9$, and 19), $N = 1.588\,579\,8$. The balance is slightly affected by the precision and rounding errors.

$Ra^T \times 10^{-6}$	Type	Branch	$\bar{K}_{sh}$	$\bar{K}_{bT}$	$\bar{K}_{bc}$	$\bar{K}_{ds}$
3.998 12	PB	B0	0.000 00	−1815.31	1818.32	−3.011 57
4.005 57	TB	B0	0.000 00	−1817.46	1820.47	−3.011 59
3.230 56	SNB	B2	0.030 87	−443.850	444.939	−1.119 76
3.234 02	PB	B2	−0.122 25	−966.289	968.084	−1.673 00
2.664 39	PB	B9	0.046 14	−336.179	337.043	−0.910 12
3.878 70	HB	B9	0.315 99	−361.611	362.724	−1.429 11
3.237 51	SNB	B8	−0.117 85	−925.701	927.427	−1.608 68
3.075 41	SNB	B4	0.029 14	−587.175	588.497	−1.351 65
3.423 88	SNB	B4	0.060 69	−668.463	669.932	−1.529 71
3.478 43	PB	B4	0.018 23	−836.903	838.673	−1.787 95
3.889 89	HB	B4	0.340 91	−491.155	492.593	−1.777 63
10.2899	SNB	B4	0.909 35	−512.067	513.382	−2.224 46
16.9172	PB	B4	2.103 55	−1404.33	1406.56	−4.325 60
18.6695	SNB	B4	1.535 46	−1229.47	1231.32	−3.391 28
4.852 90	PB	B19	0.350 14	−962.980	965.001	−2.370 60
19.9399	HB	B19	0.248 13	401.753	−400.147	−1.854 20

by $\bar{K}_{bT}$ give rise to a Feigenbaum cascade of POs at high amplitude. The weight of the terms of the energy equation at the HBs is different to that found previously^{10,11} at any of the bifurcation points. The powers $\bar{K}_{bT}$ and $\bar{K}_{bC}$ could differ one order of magnitude in the general case, and the rate of the global work done by the buoyancy forces was lower than the production of $\bar{K}$ by the shear stresses.

This study shows that a localized dynamics, preserving or not the center symmetry, is very common in binary fluid convection in the BBC, namely it is not restricted to ranges of parameters that give rise to snaking diagrams. Moreover, the inclusion of the Soret effect also maintains this steady or periodic localized dynamics.

SUPPLEMENTARY MATERIAL

See the [supplementary material](#) for the additional contour plots of the streamfunction along branches B1, B3, B12, B13, B2, and B9 (lower part) of steady solutions that are not included in the main article. Moreover, the position of the snapshots marked with crosses in the corresponding bifurcation diagrams are also included. The figures are numbered as in the article with a prefix SM.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Juan Sánchez Umbria: Conceptualization (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal). **Marta Net:** Conceptualization (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Software (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – original draft (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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